Practice Problems

Instructions: Complete each of the following as practice.

1. Show $W \leq V$ for each set W and vector space V given below.
   (a) $W = \{ax^2 + bx + c : a + b = -5c\}, \qquad V = \mathcal{P}_3(\mathbb{R})$
   (b) $W = \left\{f : \frac{d^2 f}{dx^2} + \frac{df}{dx} + 1 = 0\right\}, \qquad V = \text{Func}(\mathbb{R}, \mathbb{R})$
   (c) $W = \left\{\begin{bmatrix} a & 0 & b \\ b & a & a+b \end{bmatrix} : a, b \in \mathbb{R}\right\}, \qquad V = \mathcal{M}_{2\times 3}(\mathbb{R})$
2. Prove that $V \leq V$ for every vector space V. I.e. every vector space is a subspace of itself.
3. Prove that $\mathcal{P}_n(\mathbb{R}) \leq \mathcal{P}_{n+1}(\mathbb{R})$ for all $n \geq 0$.
4. Let A, B, and C be arbitrary vector spaces. Prove that if $A \leq B$ and $B \leq C$, then $A \leq C$.
5. For further exercises, see the following (note: this list may break with future versions of these textbooks).
   (a) Beezer NONE
   (b) Hefferon page 103 (problems 2.20 – 2.21, 2.30 – 2.34, 2.38, 2.40)
   (c) Matthews NONE